

2019

# Bottom-up Default Analysis (BuDA v3.1.1) White Paper

The Credit Research Initiative (CRI)  
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*August 5, 2019*

## ABSTRACT

Bottom-up Default Analysis (BuDA) is a credit stress testing and scenario analysis toolkit jointly developed by the Credit Research Initiative (CRI) team of National University of Singapore (NUS) and the International Monetary Fund (IMF), and operationally linked to the CRI platform (<https://www.rmicri.org>). To analyse the dynamics of credit risk under macro-financial scenarios, BuDA provides a unique framework to evaluate the credit quality of each individual firm over given scenarios, which in turn forms a bottom-up portfolio of interest. This, for example, enables IMF economists to anticipate consequences of particular macro-financial circumstances on each or a group of economies, which is fundamental to policymaking. This white paper provides the conceptual background underlying BuDA including a basic understanding of the inner workings of this scenario analysis toolkit as well as its underlying system and methods used to produce the probabilities of default (PDs).

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BuDA (v1.0) was developed by Jin-Chuan Duan of the NUS-CRI team and Weimin Miao of CriAT, a former NUS-CRI team member, in collaboration with Jorge Chan-Lau of International Monetary Fund. The NUS-CRI team provides the continual development and support of BuDA.

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\* Please cite this document in the following way: “The Credit Research Initiative of the National University of Singapore (2019), Bottom-up Default Analysis (BuDA v3.1.0) White Paper”, Accessible via [https://www.rmicri.org/en/white\\_paper/](https://www.rmicri.org/en/white_paper/).

## I. OVERVIEW

BuDA is a bottom-up approach to credit stress testing and scenario analysis. The key question within the analysis is, “How does the credit quality of an economy/sector (or a group of economies/sectors) respond to shocks to macroeconomic variables?” For example, under a prolonged recession with consecutive quarters of negative GDP growth, how badly would the economy/sector suffer?

The basic elements are individual firm PDs of listed firms which, collectively, make up the corporate sector of an economy. As mentioned, CRI produces PDs for all exchange-listed firms in the world, and the work of BuDA is to translate macroeconomic shocks into impacts on the individual firm PDs, which are then aggregated into any targeted economy/sector.

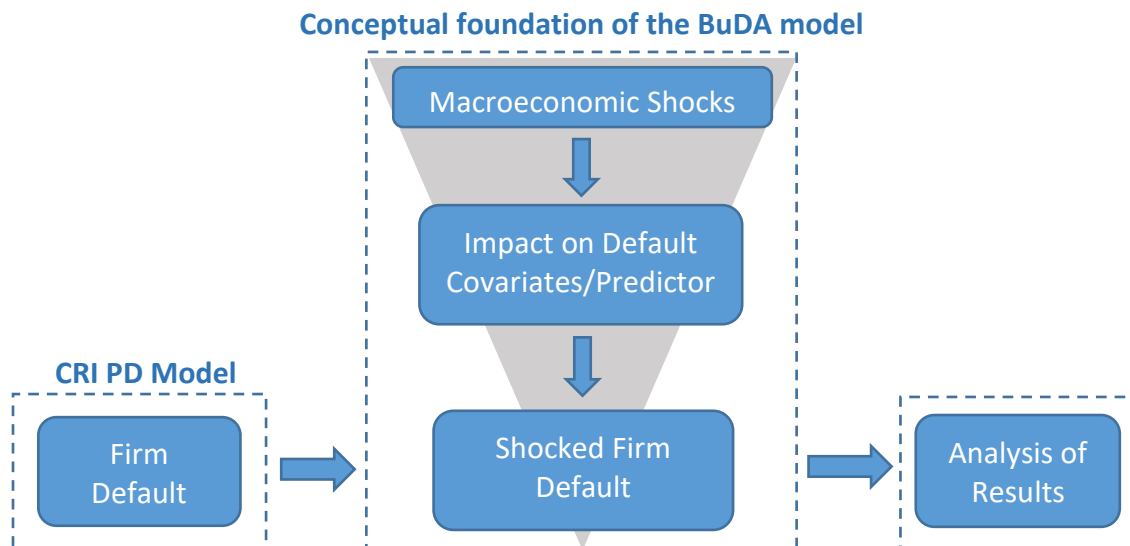


Figure 1: The Overview of BuDA Framework

Figure 1 represents the overview of BuDA model, which is based on the PD model developed by researchers affiliated with the NUS-CRI team. The BuDA model aims to assess the impact which macroeconomic scenarios may have on the CRI-PDs. For example, how would a country’s credit risk profile be affected in the event of a recession?

## II. METHODOLOGY

The BuDA framework comprises two main components. The first component is about how defaults are defined and the model for calculating CRI-PD. The second is the methodological foundation underlying the BuDA approach to macroeconomic scenario analysis.

### CRI-PD

#### What are probabilities of default (PD)?

When money changes hands, the lender is concerned about whether debt repayments, including interest and principal, will take place as originally agreed with the borrower. The failure of the latter to deliver on its obligations constitutes a default event. For example, Moody's Investor Services, a U.S.-based rating agency, defines default as one of three types of credit events: a missed or delayed disbursement of interest and/or principal; bankruptcy, administration, legal receivership, or other legal blocks to the timely payment of interest and/or principal; and a distressed exchange where the lenders receive in exchange for the original obligations a diminished obligation, or the exchange had the apparent purpose of helping the borrower avoid default.

In the CRI framework, the default event falls under at least one of three categories, namely, (1) a bankruptcy filing, (2) a delisting due to bankruptcy, or (3) a default corporate action. Table 1 and Table 2 list firm exits which are considered a default and other form of corporate exit, respectively. These default events are consistent with those of major rating agencies, and encompass similar categories of events despite differences in the legal framework across countries.

**Table 1: Firm Exits Classified as Defaults**

Default	
Action Type	Subcategory
Bankruptcy filing	Administration, Arrangement, Canadian Companies' Creditors Arrangement Act (CCAA), Chapter 7,11,15 (United States bankruptcy code), Conservatorship, Insolvency, Japanese Corporate Reorganization Law (CRL), Judicial management, Liquidation, Pre-negotiation Chapter 11, Protection, Receivership, Rehabilitation, Rehabilitation (Thailand 1997), Reorganization, Restructuring, Section 304, Supreme Court declaration, Winding up, Work out, Sued by creditor, Petition withdrawn
Delisting	Due to bankruptcy
Default corporate action	Bankruptcy, Coupon & principal payment, Coupon payment only, Debt restructuring, Interest payment, Loan payment, Principal payment, Alternative Dispute Resolution (ADR, Japan only), Declared sick (India only), Regulatory action (Taiwan only), Financial difficulty and shutdown (Taiwan only), Buyback option

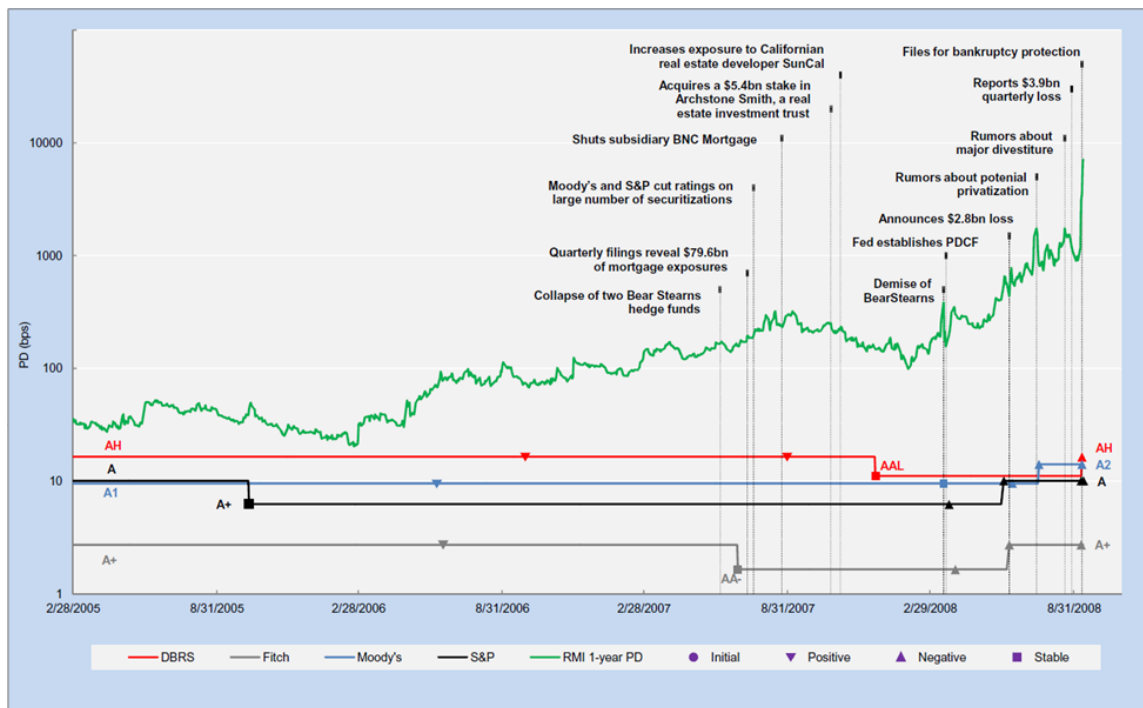
**Table 2: Firm Exits NOT Classified as Defaults**

Other Exits	
Action Type	Subcategory
Delisting	Acquired/merged, Assimilated with underlying shares, Bid price below minimum, Cancellation of listing, Failure to meet listing requirements, Failure to pay listing fees, Inactive security, Insufficient assets, Insufficient capital and surplus, Insufficient number of market makers, Issue postponed, Lack of market maker interest, Lack of public interest, Liquidated, Not current in required filings, NP/FP finished, Privatized, Reorganization, Security called for redemptions, the company's request, Scheme of arrangement, Selective capital reduction of the company, From exchange to Over-the-Counter (OTC), Privatised

A default event could trigger off several important events. Company assets and liabilities may be reorganised or liquidated, depending on circumstances. Due to the lack of sufficient assets to meet liabilities, the stock price will severely decline or be completely wiped out. Likewise, bonds and debt instruments written by the company may lose value if investors believe that the obligations cannot be fulfilled completely. Credit default swaps, essentially insurance on the company's bonds, will be triggered for pay out if they are traded.

Since losses associated with defaults can be substantial, there is interest among lenders/credit investors, analysts, financial regulators, and systemic risk supervisors in knowing how likely the default of a firm or group of firms is. The most natural default risk measure of a firm is its PD. It conveys a sense of the credit quality of a

company, with a lower PD suggesting better credit quality, whereas an average or median PD conveys an assessment of credit quality on a group of companies where averaging can be either simple or value-weighted depending on the user's purpose. Fundamentally, the companies with better credit quality are those with a better balance sheet position, liquidity and profitability, and lower stock volatility.



**Figure 2: The Collapse of Lehman Brothers**

\*Figure obtained from "A Lead-Lag Investigation of RMI PD and CRA Ratings," Global Credit Review 2012

As an illustration, let's examine the case of Lehman Brothers in Figure 2, taken from Global Credit Review, shows an analysis starting 3.5 years prior to its September 2008 bankruptcy filing under Chapter 11 of the United States Bankruptcy Code. Through this period, the CRI 1-year PD rose steadily from less than 100 basis points (bps<sup>1</sup>) to almost 10,000 bps in September, 2008. The vertical axis is in the log scale with each tick amounting to 10 times in PDs. On the same graph, the credit ratings over the same period issued by major credit rating agencies (CRAs) such as S&P, Moody's's, etc. are plotted. The letter ratings offered by the CRAs have been converted into probabilities using their respective reported historical realized annual default rates. The graph clearly

<sup>1</sup> 100 basis points = 1%.

demonstrates that Lehman Brothers' risk of default was according to the CRI PD much higher than the credit risk assessments reached by CRAs at that time.

The PD tells us how likely it is for a firm to default over a given period in future. Naturally, we are concerned when a company's PD rises beyond its historical levels or the level at which we entered into a debt or credit risk position. But our focus may go beyond a single firm. If we broaden our perspective beyond a single company, we can also study current levels of the overall PDs of specific industrial sectors or the broader economy, and their potential behaviour in response to changing economic conditions. For example, consider a macroeconomic recession scenario where GDP falls continually. This may cause the PDs of many companies to rise together, increasing the overall PD levels in the economy and raise systemic risk concerns. The BuDA approach, covered in detail in the later section, can identify this type of situations, helping regulators, lenders, economists, and market observers analyse and manage risks more adequately.

In comparison to most other sources of credit risk information, CRI-PDs are more responsive, being updated daily. The CRI-PD model employs a cutting-edge default econometrics with its content fully disclosed. The CRI-PD model's implementation is refined over time, and its recalibration is performed monthly to capture potential changes in model parameters. The model incorporates market data such as the stock market performance, and prevailing interest rate, among other information.

## The CRI-PD model

In this section, we cover the key features of the CRI-PD model with the aim of conveying the intuition behind how it works. Technical details are available in the later section which the reader can skip if interested only in running the model.

### *Data*

Estimating PDs is particularly challenging and difficult in part because we cannot observe their values directly. We only observe whether a firm defaults at a particular point of time, but defaults happen rather infrequently. Since there is no unambiguous direct measurement of PD, one can only hope to empirically estimate PDs by applying a model on a large sample of historical records on the default/survival status of firms along with their relevant and observable attributes.

Therefore, we begin by observing actual realized defaults and other corporate exits, and identifying the circumstances under which they occur. Variables which are associated with or have predictive properties of default outcomes are called *default covariates/predictors*. Empirical studies have identified several covariates/predictors which seem particularly associated with the occurrence of defaults, which are incorporated into the CRI-PD model. Table 3 provides a list of the covariates and a brief description. The covariates/predictors fall under two major categories, economy-wide risk factors<sup>2</sup>, and firm-specific attributes. The first category, as the name indicates, includes covariates/predictors that tend to affect all firms in the economy and are essentially common risk factors.

**Table 3: Covariates Used in Default Prediction**

Covariate/predictor	Brief Description
<b>Economy-wide risk factors</b>	
Stock index	Trailing 1-year return on the stock market. A poor stock market performance is generally associated with weaker firm performance and higher probability of firm default.
Short-term interest rate	Yield on 3-month government bills. Generally speaking, if borrowing costs are higher, firms may face more funding constraints and may be more likely to default in the short-term. In the longer horizon, a higher interest rate may indicate positive economic growth and hence lower solvency risk.
Aggregate DTD	Financial aggregate DTD is median DTD of financial firms in each economy inclusive of those foreign financial firms whose primary stock exchange is in this economy. Non-Financial aggregate DTD is median DTD of non-financial firms in each economy inclusive of those foreign non-financial firms whose primary stock exchange is in this economy.

<sup>2</sup> In the original paper, Duan, Sun and Wang (2012), economy-wide risk factors were referred to as macroeconomic risk factors. However, we reserve this term for scenario analysis/stress testing variables which we discuss in BuDA section.



(Table 3 – continue)

Covariate/predictor	Brief Description
<b>Firm-specific attributes</b>	
Distance-to-default <ul style="list-style-type: none"> <li>• Level</li> <li>• Trend</li> </ul>	Volatility-adjusted leverage measure based on Merton (1974), which is the logarithm of the ratio between the market value of a firm's assets and its liabilities, scaled by the asset volatility. A smaller DTD increases the likelihood of default. CRI uses a modified DTD measure which we will discuss later.
Liquidity <ul style="list-style-type: none"> <li>• Level</li> <li>• Trend</li> </ul>	Logarithm of the ratio of a firm's sum of cash and short-term investments to its total assets for financial firms. Logarithm of the ratio of current asset to current liability for non-financial firms. Higher liquidity is more beneficial for the firm.
Profitability <ul style="list-style-type: none"> <li>• Level</li> <li>• Trend</li> </ul>	Ratio of each firm's net income to total assets. Higher profitability is more beneficial for the firm.
Relative size <ul style="list-style-type: none"> <li>• Level</li> <li>• Trend</li> </ul>	Logarithm of the ratio of each firm's market capitalization to the economy's median market capitalization over the past year. Loosely speaking, bigger firms tend to have fewer defaults, although we sometimes observe the opposite.
Relative market-to-book ratio	Ratio of each firm's market value (market capitalization plus total book value of liabilities) to its book value (total book value of assets) relative to that of the economy. It captures the misvaluation or growth opportunity effect.
Idiosyncratic volatility	The variation of firm returns which cannot be attributed to the stock market index, using daily data from the past 1 year. Firms with higher idiosyncratic volatility tend to have more variable cash flows and a higher probability of bankruptcy.

CRI aims to provide the credit risk data of every country/economy globally. By far, major effort is put into collecting data on covariates/predictors, defaults and other corporate exits on over 67,000 firms in 128 economies across all continents. Among these firms, over 34,000 firms are currently active, and on an ongoing basis, effort has been committed to monitoring these firms and collecting the relevant data. Market-based data such as stock prices and interest rates are updated daily, while data from financial statements are checked daily and updated once available. The main sources of data are Thomson Reuters Datastream and the Bloomberg Data License Back Office Product. This rich information has currently power the BuDA toolkit.

Of the 128 economies, 88 of them have national stock exchanges, and for each of those, a specific, representative stock index and a short-term interest rate are chosen. For the remaining economies, CRI covers the companies which are domiciled in the economy economy but quoted on a foreign exchange, mostly because those economies do not have a stock exchange.

Financial statements and market data for individual firms serve to compute the remaining firm-specific covariates/predictors, namely, DTD, liquidity, profitability, relative size, relative market-to-book ratio, and idiosyncratic volatility. Calculating the value of the covariates/predictors is straightforward, except for the Distance-to-Default, for which we provide a more in-depth explanation as follows.

#### Level and trend variables

To increase the predictive accuracy of the PD model, it is helpful to include the trend value, in addition to level values, for some of the firm-specific attributes. Level covariates are computed as the one-year moving average of the measure, and the trend is computed as the current value of the measure minus the one-year moving average of the measure. Considering Figure 3 below, Firm 1's DTD has been falling over consecutive periods, while Firm 2 has been rising. Even though they both currently sit at the same point, the statistical trends suggest that in the next period, it is quite likely that the DTD of Firm 1 will fall below Firm 2. Duan et al (2012) found that including trend significantly improves the predictive power of the model for short-term horizons.

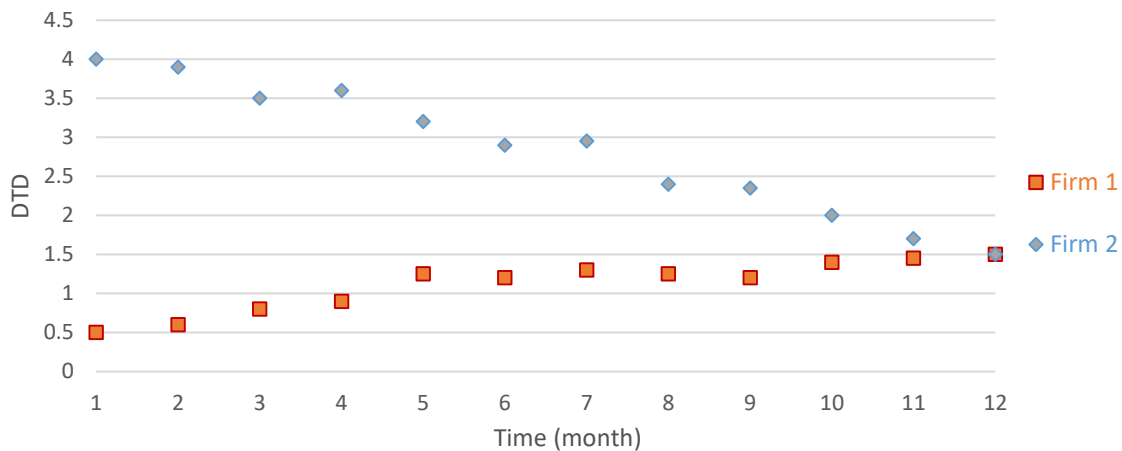


Figure 3: DTD Trend

### Distance-to-Default (DTD)

KMV, now part of Moody’s Analytics, first introduced the commercial usage of DTD, which serves as the foundation of its Expected Default Frequency model. Empirical studies have shown that DTD is among the best predictors of default.<sup>3</sup> CRI uses DTD as one of its inputs in its PD model. While conceptually similar to KMV’s DTD, there are differences in the calculation of the CRI’s DTD to incorporate liabilities more holistically and make it applicable to financial firms, as described later in this section.

Table 4: A Simplified DTD Example

	Weak Firm	Strong Firm
Assets (\$) (current value)	110	200
Liabilities (\$) (promised amount in the future)	100	100
Volatility of Assets	20%	20%
Distance-to-Default (simplified)	$10\% / 20\% = 0.5$	$100\% / 20\% = 5$

For now, to facilitate our description, consider a simplified example of two firms in

Table 4, financially weak and strong firms. Which firm is more likely to default? Let’s take a look at Weak Firm. It has assets of \$110 and liabilities of \$100. Currently, the value of

<sup>3</sup> Studies which have used DTD as a default covariate/predictor include Crosbie and Bohn (2001), Vassalou and Xing (2004), Duffie, Saita and Wang (2007), Bharath and Shumway (2008), and Duan, Sun and Wang (2012), to name a few.

the assets covers the value of the liabilities (after discounting) adequately. The value of assets, however, fluctuates over time due to several reasons. Among them, adverse business conditions or ineffective collections could reduce the amount of account receivables. Changing business circumstances may render some of its fixed assets irrelevant and devalued. Securities held by the firm, such as bonds or stocks may lose value. Over time, there is no guarantee that the value of assets will remain at or exceed \$100 at the time of liabilities due so that all liabilities can be fully met. This lies behind the option-theoretical basis for DTD in the Merton (1974) model, where the liabilities, i.e. the promised payment, serves as the strike price of a call option and the event in which the promised payment is not fully met amounts to the call option finishing out of the money. The uncertainty over how much the asset will be worth is summarized in “volatility of assets”, which is 20% in this example.

DTD attempts to factor in the volatility of asset values. It measures how much headroom assets hold over liabilities per unit of asset volatility. For Weak Firm, assets exceed liabilities by 10 percent, and the volatility of assets is 20 percent, which yields a DTD value of 0.5 (10 percent / 20 percent). In other words, excess assets over liabilities are sufficient to buffer a 0.5 standard deviation shock to the value of its assets. Likewise, for Strong Firm, the DTD is 5 (100 percent / 20 percent) which is ten times as high as that of Weak Firm. Strong Firm’s assets can absorb a shock of up to 5 times the standard deviation of its asset value. Therefore, Strong Firm is “further” away from default than Weak Firm, and correspondingly, it will have lower PD.

The above example is extremely simplified, so several clarifications are in order. First, we are looking at the market value of assets, not the book value of assets. Book values were recorded at point of entry and can be quite outdated, and fail to reflect latest valuations. Second, the actual formula for DTD used in practice such as the CRI-PD model is more complicated, involves the use of logarithm and square root scaling to the appropriate time horizon. For completeness, we show below the DTD formula at time  $t$  adopted by the CRI system for a firm whose time- $t$  asset value is  $V_t$ , liabilities due at time  $T$  is  $L$ , and volatility rate is  $\sigma$ .<sup>4</sup> However, our aim here is to convey the intuition rather than to cover the finer points.

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<sup>4</sup>We have purposely left out the risk-adjusted drift term, i.e.  $(\mu - \frac{\sigma^2}{2})(T - t)$ , in the original DTD formula to obtain a practically more informative DTD. This is because one cannot estimate  $\mu$  with a reasonable precision due to the high noise-to-signal ratio inherent in typical daily stock returns, a well-known fact in the financial time series literature. For a more complete discussion on the DTD formula, we refer readers to Duan and Wang (2012).

$$DTD_t = \frac{\ln\left(\frac{V_t}{L}\right)}{\sigma\sqrt{T-t}}$$

### Asset and liabilities in DTD

Let's dig one level deeper. How can we estimate the market value of assets and the promised payment, i.e., some sort of book value of liabilities for DTD purposes given the complex capital structure in reality?

To start, one must first determine what  $(T - t)$  to use in practice. For no apparent reasons, the common practice has settled on setting it to one year. But obviously, liabilities for typical firms will scatter a wide maturity spectrum, and hence there is a need to apply some *ad hoc* but sensible adjustment to turn liabilities into a pseudo promised payment (referred to as default point hereafter) in 1-year time. Now, since DTD looks ahead over a 1-year horizon, we can arguably count short-term debts (due within a year) directly in the default point. However, it is only reasonable to subject long-term debts (due beyond one year) to a haircut in order to conform to the 1-year horizon. The practice advanced by KMV and adopted widely in the credit literature is to haircut long-term liabilities by 50% before adding them to the default point, reflecting the fact that long-term debts are due later than one year.

The CRI model has incorporated an *additional* component into the default point. Financial firms are notorious for being difficult to model, since the majority of their liabilities are neither in the short-term nor in the long-term debt category. We refer to the additional and rather large amount of liabilities as *other liabilities*. Within this category, a predominant component for banks is, of course, customer deposits which make up the bulk of a bank's total liabilities. To substantiate the point, it is not uncommon for banks to be leveraged 10-20 times, with the majority of liabilities falling under – you guessed it – *other liabilities* as deposits. For insurance companies, policy obligations will mainly constitute other liabilities.

Prior work in the credit literature typically pays scant attention to the role and impact of other liabilities, which are not particularly large in size vis-a-vis short-term and long-term debts for most non-financial firms. When financial firms are conventionally excluded from

default analysis, ignoring other liabilities becomes lesser of an issue.<sup>5</sup> In contrast to most studies, the CRI model, following the approach of Duan, Sun and Wang (2012), directly factors in other liabilities by adding to the default point formula a fraction of other liabilities where the haircut rate is treated as an unknown parameter to be estimated along with other model parameters. One way to think about this issue is that customer deposits represent obligations, a portion of which may be withdrawn on short notice while others may stay with the bank for a long term. In order to subject them to the same 1-year horizon, haircutting is sensible, but the magnitude should not be arbitrary. Setting the haircut rate as an unknown parameter and letting the data inform us seems the most logical way of determining its value. Overall, the modification to the default point formula of KMV given below enables us to expand the scope and produce PDs for financial firms in addition to non-financial firms.

$$\text{DTD Liabilities} = (\text{Short-Term Liabilities}) + 0.5 (\text{Long-Term Liabilities}) + \delta (\text{Other Liabilities})$$

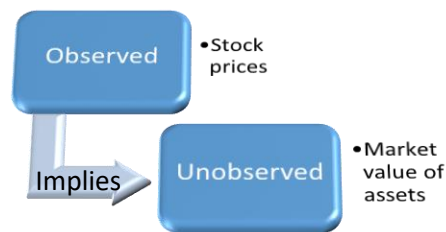
Moving on, our next challenge is to estimate the market value of assets. Because market values cannot be observed directly, we must rely on indirect methods to measure it. To draw an analogy, we cannot see the healthiness of a person's heart directly, not unless we surgically remove and comb through it for blockages and clots. We are not completely helpless, however as we could instead measure his cholesterol level, we could check his blood pressure, obesity level, run an ECG and interview his lifestyle habits to draw a fairly accurate diagnosis of his heart condition, all the while without looking directly at his heart (or removing it). In the same way, we can assess the market value of a firm's assets by looking at its stock price. If the stock price rises, it should mean that the market value of assets rises. Why? Because stock securities are in fact call option-like claims on assets. More precisely, stock securities are claims on the residual value of assets after creditors (debt holders) have been paid off.

So, there is information pertaining to market value of assets within stock prices, and all we need is a suitable technique to extract it. For those of us who are familiar with options, stocks are essentially call options on the market value of assets, with the liabilities (default point in our adopted jargon) being the strike price. Consequently, exploiting this fact, we

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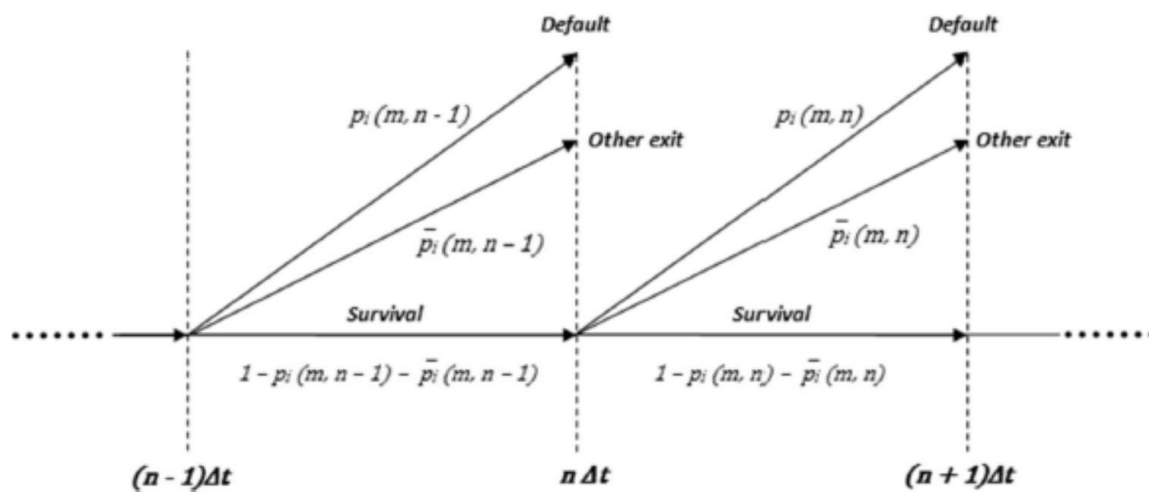
<sup>5</sup> At odd with the typical finding, Campbell, Hilscher and Szilagyi (2008) concluded that DTD does not help with default prediction. According to Zou (2016), their conclusion was due to the inclusion of financial firms in the sample while leaving other liabilities untreated. Zou (2016) shows that using either the DTD with the KMV default point formula on the sample excluding financial firms or the DTD with the revised default point formula factoring in other liabilities will reverse the conclusion of Campbell, Hilscher and Szilagyi (2008).

can infer market values of the assets from stock prices by inverting the option pricing formula. For finer points and references on the estimation method, we refer readers to Duan and Wang (2012). If this sounds too technical, the main takeaway here is that market values of assets, while not observable, can be extracted from stock prices. It is like looking for the smoke to find out where the fire is.



### *Modelling the Default Process*

In order to utilize the data collected, we must translate the covariates/predictors into default and other-exit probabilities, keeping in mind that we observe only the actual defaults and other forms of corporate exits. Imagine for a moment that time can be split up into discrete periods. In each of these periods, one of three events may occur. First, the firm survives and continues as a viable entity in the next period. Second, the firm may default, in which case a “default” event is observed. Third, the firm may be delisted or merged with another firm, in which case we observe an “other-exit” event. We include this third category of events because they are a key component to measuring survival likelihood and vital to the estimation of default probabilities free of survival biases in a multi-period environment. In some economies as documented in Duan, Sun and Wang (2012) and the NUS-RMI CRI Technical Report (2017), exits due to reasons other than default can be up to ten times as many as exits due to default.



An attentive reader may realise that we are trying to associate the aforementioned covariates/predictors with the period by period probability of default. If the situation for the firm is unfavourable, for example, DTD is low, and liquidity and profitability are poor, this translates into a higher probability for the firm to default. However, firms may still survive when the situation is bad. Being a matter of chance, the firm may or may not actually default in the current period. They are simply *more likely* to default. If the bad situation persists for a period of time, chances are that the firm will default eventually. In this manner, the evolution of the covariates/predictors over time may be related to the actual realised defaults.

This intuition can be formalised as a Poisson process, which is a common method for modelling the occurrence of a rare event (default or other exit in this case) as time evolves. The CRI model, based on Duan, Sun and Wang (2012), takes in the set of covariates/predictors and combines them into time-varying forward *default intensities*, which reflect default and other-exit events being generated via a pair of Poisson processes for each firm, and their intensities are correlated through dependency among covariates/predictors. Across firms, different pairs of intensities may also be correlated via their covariates/predictors.



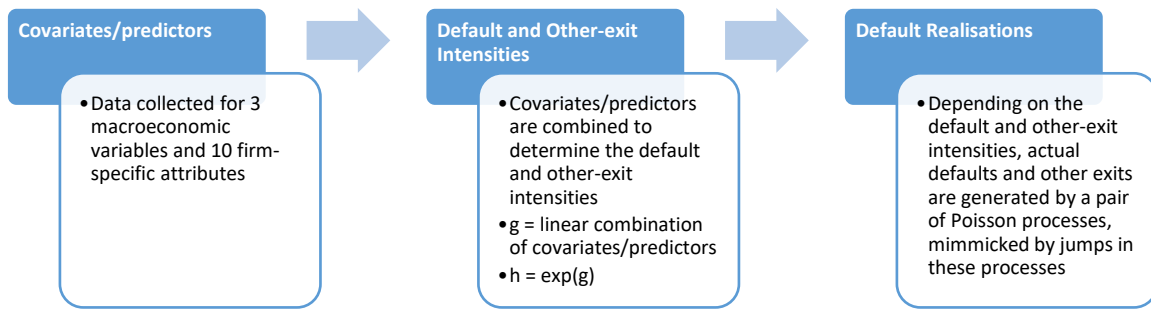


Figure 4: Modelling of Defaults and Other Exits Using a Pair of Poisson Processes

### Estimation

How do we combine the covariates/predictors into the forward default/other-exit intensity? Until now, we have not specified exactly how this is done. A straightforward way to combine the information coming from each of the covariates/predictors is simply to add them up, in other words, compute:

$$\begin{aligned}
 g = & \beta_0 + \beta_1 (\text{Stock index}) + \beta_2 (\text{Short-term interest rate}) + \beta_3 (\text{Aggregate DTD}^6) \\
 & + \beta_4 (\text{Distance-to-default Level}) + \beta_5 (\text{Liquidity Level}^7) + \beta_6 (\text{Profitability Level}) \\
 & + \beta_7 (\text{Relative size Level}) + \beta_8 (\text{Relative market-to-book ratio}) + \beta_9 (\text{Idiosyncratic volatility}) \\
 & + \beta_{10} (\text{Distance-to-default Trend}) + \beta_{11} (\text{Liquidity Trend}) + \beta_{12} (\text{Profitability Trend}) \\
 & + \beta_{13} (\text{Relative size Trend}).
 \end{aligned}$$

Next, having a negative default intensity does not make sense, since a presently surviving firm can only default or survive in the next period; it cannot “anti-default” as there is no such interpretation. Therefore, the default intensity is set to  $h = \exp(g)$ , or taking an exponential of the weighted sum of individual covariates/predictors to ensure that the intensity is positive.

Estimating the model is then a matter of choosing the parameters  $\beta_0, \beta_1, \beta_2, \dots, \beta_{13}$  so that the model best matches the data (observations of actual defaults/other exits).

<sup>6</sup>  $\beta_3$  is different for financial firms vs. non-financial firms, as the aggregate DTD measures are different for the two groups. See NUS-RMI Staff, 2017, Addendum 9 to the CRI Technical Report (Version: 2017, Update 1), CRI website.

<sup>7</sup>  $\beta_5$  and  $\beta_{11}$  are different for financial firms vs. non-financial firms, as the liquidity measures for the two groups are defined differently. See NUS-RMI Staff, 2017, Addendum 4 to the CRI Technical Report (Version: 2017, Update 1), CRI website.

Briefly, this involves writing the likelihood of the model given the data, and maximizing it over the parameters. To grasp the intuition, consider the parameter  $\beta_6$  for profitability. Since defaults tend to occur when profitability is low, we expect  $\beta_6$  to be negative. A positive  $\beta_6$  means that the default intensity increases with profitability, and in reality disagrees with what the data says. In other words, it is not very likely for  $\beta_6$  to be positive. The optimisation method picks  $\beta_6$  and all the other  $\beta$ s simultaneously so that they best agree with what we see in the actual defaults/other exits. A full statistical treatment would be too involved here, but the key takeaway is that the parameters are selected to explain the default data as much as possible.

Recall that CRI produces a term structure of PDs from 1 month up to 5 years. To achieve this, we need to estimate not only a single set of parameters  $\beta_0, \beta_1, \beta_2, \dots, \beta_{13}$ , but an entire term structure of them up to 5 years into the future. Intuitively, it might sound infeasible or impossible to estimate the parameters for each and every time point into the future up to 5 years. But it is actually doable via a decomposability result established in Duan, Sun and Wang (2012). Naturally, we would expect adjacent horizons to share similar parameter values, and hence we can assume that the term structure of the parameters follows a particular class of curves. So using  $\beta_6$  as an example again, we assume that its values across the various forward-looking horizons can be described as a curve, then instead of estimating each and every  $\beta_6$ , we estimate the shape of the curve which turns out to be computationally more challenging but intuitively more desirable. Apart from smoothing the parameter estimates, an added benefit is that the parameter curve allows for easy extrapolation into longer horizons for which data may not even exist. Exactly which shape this curve takes is a matter of choice. For example, we may simply decide to fit a quadratic (polynomial) curve, but this form may perform poorly. Duan, Sun and Wang (2012) deployed a Nelson–Siegel curve and the CRI model followed. While less widely known, the Nelson-Siegel curve is adequately simple and works well. This approach is very similar to how the term structure of bond yields is modelled.

### *Calibration Groups*

Earlier we mentioned that the CRI coverage spans over 67,000 firms. With such an extensive diversity, it is only natural to expect that the parameters  $\beta_0, \beta_1, \beta_2, \dots, \beta_{13}$  differ across different economies of firms. But at the same time, one must strike a balance with the need to have a sufficient number of defaults in the sample to conduct model estimation/calibration. The CRI system thus deploys calibration groups by putting together firms from different economies that are expected to be more similar than not.

Currently, Canada and the US belong to the North America calibration group, and the developed economies of Asia-Pacific (Australia, Hong Kong, Japan, Singapore, South Korea, Taiwan and New Zealand) form another calibration group. China and India, the two major emerging economies of Asia Pacific are each calibrated as individual groups. All the European countries covered by the CRI are in a single calibration group. The other emerging economies of Asia Pacific, Latin America, Middle-East and Africa form the “emerging markets” calibration group.

As a general rule, all economies in the same calibration group share the same coefficients for all variables except for the short-term risk-free interest rate variable<sup>8</sup>. The short-term interest rate variable is entered as the current value minus the historical month-end mean in order to reflect the contemporary change relative to the historical average. Its coefficient is allowed to vary across economies, because different economies with different currencies may have different dependencies on their interest rates, the levels of which can also differ significantly across economies. The Euro zone deserves a special explanation, where all countries after joining the Euro zone begin to use German short-term interest rate.

### *Prediction Accuracy*

The CRI conducts tests to ascertain that the PDs are informative of potential defaults. A popular standard in assessing the discriminatory power of a rating system is to use Accuracy Ratio (AR). The intuition behind AR is that if firms with high PD are indeed those that actually default, then AR is high. In other words, the distressed firms have been properly discriminated or distinguished from the safe firms. On the other hand, if PD levels have little to do with defaults, then AR is low. AR ranges from 0 to 1, with 0 indicating a completely uninformative rating system, and 1 a perfect system. The use of AR has been discussed, for example, in a paper on rating validation prepared by the Basel Committee on Banking Supervision (BCBS, 2005b).

Table 5 reports the ARs for various prediction horizons and economies. For example, the procedure for 1-year AR calculation is as follows: first, we calibrate the parameters using the full data set (in the tables below, this is until March 2017). Next, standing at a particular point in time, say 31 December 2000, we extract the PD forecasts 1 year ahead for all firms based on these parameters as well as the actual defaults that occurred in

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<sup>8</sup> As exceptions, Eurozone uses German interest rate owing to their economic integration. In addition, Indonesia has its own coefficient on relative size.

2001. We subsequently pool the PD forecasts across all time spots for this economy and compare them against the actual defaults so as to get the 1-year AR.

**Table 5: Accuracy Ratios for the CRI-PD Model**

Economy	AR			
	1mth	1yr	2yr	5yr
Australia	0.83246	0.67209	0.56095	0.39794
Brazil	0.87197	0.80106	0.71095	0.49877
Canada	0.95304	0.82627	0.70260	0.49930
China	0.70164	0.69218	0.65805	0.53890
Germany	0.86694	0.69298	0.57753	0.46766
Denmark	0.82046	0.76551	0.61542	0.53745
France	0.85554	0.72841	0.63541	0.55865
Hong Kong	0.77080	0.53696	0.42181	0.23816
India	0.72501	0.65636	0.60090	0.47297
Indonesia	0.73299	0.69022	0.60649	0.41965
Italy	0.87879	0.82573	0.66153	0.46295
Japan	0.91234	0.85442	0.79959	0.66773
Malaysia	0.84062	0.78735	0.71994	0.53608
Mexico	0.82134	0.79538	0.73614	0.56559
Netherlands	0.88257	0.83564	0.68133	0.55146
Norway	0.95316	0.82900	0.60700	0.29491
Philippines	0.72890	0.65725	0.64384	0.57070
Poland	0.87899	0.75093	0.59734	0.34276
Russian Federation	0.79778	0.42557	0.19024	0.083178
Singapore	0.81505	0.71364	0.54761	0.32252
South Africa	0.92794	0.85689	0.74132	0.46368
South Korea	0.87184	0.73760	0.66442	0.55799
Sweden	0.90945	0.79508	0.68702	0.41611
Taiwan	0.87882	0.77457	0.70024	0.61224
Thailand	0.81878	0.78254	0.73483	0.61134
United Kingdom	0.88910	0.77209	0.62690	0.43333
United States	0.94433	0.83615	0.72316	0.53831
Developed Asia-Pacific	0.86693	0.74847	0.66370	0.53063
Emerging MKT	0.82510	0.77456	0.70990	0.55807
Europe	0.87762	0.75037	0.61715	0.45191
North America	0.94531	0.83533	0.72152	0.53539

\*The table is taken from addendum 2 to the CRI technical report 2017 update 1.

The model is able to achieve strong AR results mostly greater than 0.80 at the one-month horizon, with stronger results for developed economies. At 1-year, ARs are mostly healthy and above 0.70. There is a drop in AR at the 2-year and 5-year horizons, but this is to be expected as we move further on in the term structure.

The ARs in some emerging market economies such as India, Indonesia, and the Philippines are noticeably weaker than the results in the developed economies. This can be due to a number of issues. The quality of data is worse in emerging markets, in terms of availability

and data errors. This may be due to lower reporting and auditing standards. Also, variable selection is likely to play a more important role in emerging markets. The variables were selected based on the predictive power in the US. Performing variable selections specific to the calibration group are expected to improve predictive accuracy, especially in emerging market economies. Finally, there could be structural differences in how defaults and bankruptcies occur in emerging market economies. If the judicial system is weak and there are no repercussions for default, firms may be more prone to default.

Previously, China's AR for 1-year PD was 57%, but has improved to 67% with the introduction of structural break estimation. The structural break occurs in December 2004, and we allow the coefficients for DTD level and the intercept to be different after this break. However, we incorporate a modification to standard structural break estimation. Instead of a sudden change in these coefficients, we allow the parameters to change smoothly<sup>9</sup> into the new parameters over time, reflecting gradual changes rather than being brought about by a sudden shift in economic structure. Currently, we model the structural break by a step function allowing for different rates of transition to and away from the break point. The treatment is the same for the intercept term and the coefficient for the DTD level, but the transition rates are different. The performance improvement in terms of the AR under the new specification is not pronounced. The AR for 1-year CRI PD increase from 67% to 69%. However, going beyond mere risk ranking firms reveals a more meaningful performance improvement.

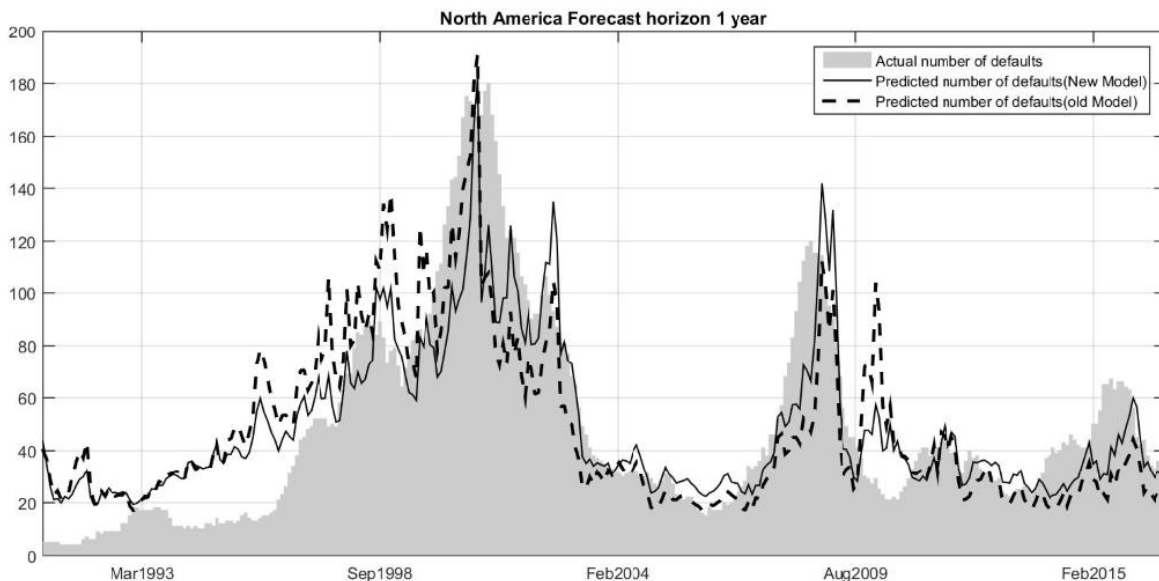
The North America calibration group (the US and Canada) has incorporated two specific changes. First, a dummy variable on the intercept for financial firms is included to account for differences that have not been duly reflected through over covariates. Second, a structural break, which is treated as an impulse response, is applied to this financial-sector intercept dummy to address the change in September 2008 after Lehman Brothers' default. After incorporating the two specific treatments, the AR for the 1-year CRI PD (calibrated in April 2018) increases from 84% to 86%. Although the increase in the AR is not pronounced, the revised model has clearly delivered a meaningful improvement in predicting default rates for the North America calibration group.

ARs are good for determining if PDs rank the firms correctly according to their relative default risks. For example, a riskier firm should have a higher PD. There is a slight catch, however. Theoretically speaking, even if we multiplied all our PDs by a factor of say 2 or

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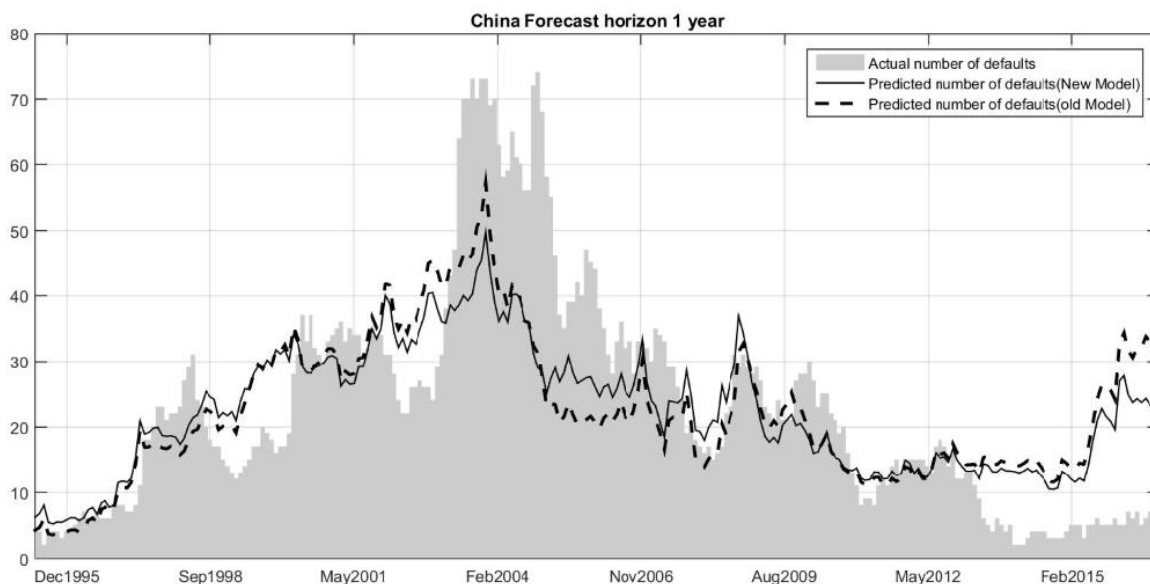
<sup>9</sup> This is implemented using a logistic function which starts with the old set of parameters and tends to the new set of parameters as time goes on. See NUS-RMI CRI Technical Report (2017).

3, the accuracy ratio would remain unchanged. Therefore, we also want to find out if the actual PD level, say a PD of 5%, actually reflects a 5% default risk. In other words, we want to assess the goodness of fit of the PDs. This is done by comparing the predicted and actual number of defaults at the aggregate level over time. Standing at a particular time point, we ask how many firms are expected to default in the next year, which we can compute using the forward PDs that have already been estimated at that point. We then compare this number with how many firms actually defaulted in the next year. Figure 5 shows this for the North America group and Figure 6 for China<sup>10</sup>. The predicted levels track the actual levels quite nicely, bearing in mind that the numbers are predicted in advance of the actual defaults. Most of the other economies exhibit a similar pattern.



**Figure 5: Predicted and Actual Defaults for United States**

<sup>10</sup> Figure 5 and Figure 6 are from CRI staff calculations.



**Figure 6: Predicted and Actual Defaults for China**

### Alternatives for CRI-PD

We now briefly mention a few alternatives which may be used to assess the credit quality of a company.

Letter credit ratings issued by credit rating agencies (CRAs) such as Standard & Poor's, Moody's Investor Services, and Fitch Ratings are possibly among the most widely used. For example, the AAA/Aaa grade is the highest possible rating and represents companies with extremely low credit risks. The opposite is true for companies rated CCC. While popular and easy to understand, criticisms of such letter credit ratings have arisen in particular after the financial crisis of 2007-2009. One topic is the lack of responsiveness to changes to the company's circumstances. Indeed, Lehman Brothers was rated as an investment grade company with at least an A rating by the big three rating agencies up until mid-September right before its bankruptcy. Another criticism is that CRAs have a profit incentive in the companies which they rate, leading to potential conflicts of interest, as noted in The Financial Crisis Inquiry Report of 2011 prepared by the Financial Crisis Inquiry Commission.

In comparison, CRI's PDs are more responsive, being updated daily and recalibrated monthly. The model incorporates market data such as the company's distance-to-default

(DTD), the stock market performance, and prevailing interest rate, among other information. CRI's PDs are provided as a public good and distributed as a free service with the intention of promoting research and development in credit risk.

More closely related to CRI's PDs are Moody's Expected Default Frequency (EDF) database and Kamakura's PDs, but neither of them intends to measure the probability that a firm will default over a specified period of time. According to available information, Moody's EDFs primarily incorporate a distance-to-default measure but without further technical details on its model, whereas the Kamakura's method is unclear to us. Both Moody's EDFs and Kamakura's PDs are paid services. CRI's PDs, on the other hand, incorporates not only a DTD measure, but also a range of macroeconomic and firm-specific attributes as explained earlier, and the methodology is fully disclosed to the public.

#### *Summary of CRI-PD*

PDs tell us how likely it is for a firm to default over a given period in future, and convey a more granular sense of the credit quality of a company. We can use PD to study a range of topics, including questions pertaining to financial stability. The PDs produced by the CRI have the advantages of global coverage, a term structure of up to 5 years, and they are free with the methodology fully disclosed. Being a non-profit set up, it does not bear an inherent conflict of interest with profit incentives.

The CRI model uses the Poisson process to model survival, and is calibrated against actual defaults and other exits. The default and other-exit intensities for the Poisson processes are linked to 3 economy-wide risk factors and 10 firm-specific attributes, namely, stock index, short-term interest rate, Aggregate DTD, relative market-to-book ratio, idiosyncratic volatility, DTD, liquidity, profitability, and relative size, with the latter 4 having both level and trend variables. DTD is particularly helpful for predicting defaults, and our modification of incorporating customer deposits and insurance policy obligations (other liabilities) makes it applicable to financial firms.



## BuDA

Equipped with a basic but workable understanding of CRI's PDs, we will explore the conceptual underpinnings of the bottom-up default analysis in this section. Implementation details and operation of the toolkit will be left for the additional document, BuDA operational guide.

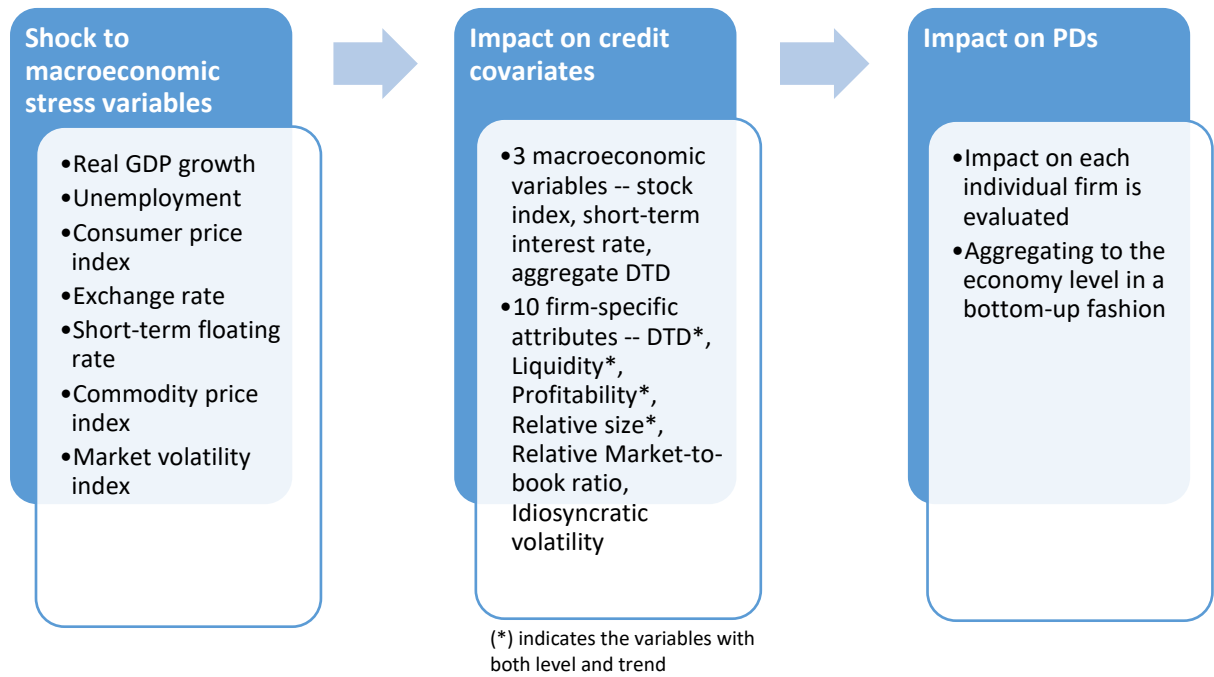
### What is Bottom-up Default Analysis (BuDA)?

BuDA is an approach to credit stress testing and scenario analysis that relates shocks to macroeconomic stress variables, such as GDP, unemployment, inflation and others, to the credit quality of the economy. For example, an economist might be interested in analysing the health of the economy when a prescribed severe recession, i.e. consecutive falls in GDP, occurs. BuDA translates the specified fall in GDP into an impact on the default covariates/predictors discussed in the previous section, which in turn determines the impact to the PDs of the targeted firms, see Figure 7. In other words, starting with the macroeconomic dimensions which an economist might be concerned with, BuDA interprets these shocks in terms of their corresponding effect on credit quality. BuDA can be useful to regulatory authorities, central banks, and commercial/investment banks for increased awareness and understanding of risks.<sup>11</sup>

As one might expect, the macroeconomic variables interesting an economist may not coincide neatly with the default covariates/predictors – 3 economy-wide risk factors and 10 firm-specific attributes which are used in the PD calculation, covered in the previous section. The idea is therefore to map the impact of the macroeconomic shocks to these default covariates/predictors, and in turn measure the follow-on impact on PDs. Let's use some math, and through which we can convey the intuition more concisely.

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<sup>11</sup> The BuDA tool caters for user-supplied stress variables other than the 7 listed here. It also caters for aggregation to a user-supplied portfolio instead of pre-specified economies.



**Figure 7: Overview of BuDA Approach**

### Equation 1: Stress Testing Regressions

Change in economy-wide risk
stress variables
Lagged economy-wide

$$\Delta X_{m,t} = \beta_{m,0}^X + \sum_{k=1}^n \beta_{m,k}^X Z_{k,t} + \gamma_{m,1}^X X_{m,t-1} + \gamma_{m,2}^X X_{m,t-2} + \varepsilon_{m,t}^X$$

$$\Delta \bar{Y}_{i,j,t} = \beta_{i,j,0}^Y + \sum_{k=1}^n \beta_{i,j,k}^Y Z_{k,t} + \gamma_{i,j,1}^Y \bar{Y}_{i,j,t-1} + \gamma_{i,j,2}^Y \bar{Y}_{i,j,t-2} + \varepsilon_{i,j,t}^Y$$

Change in firm-specific attributes
stress variables
Lagged firm-specific

Random error terms

As illustrated in the equations, changes to the default covariates/predictors are driven by the macroeconomic stress variables. Since some of the covariates/predictors exhibit persistence, we include lag terms to take care of autocorrelation. For those who are familiar with autoregressive systems, the model implemented above is an AR(2) model, i.e. 2 lags. This linkage to macroeconomic stress variables of interest affords flexibility of choice and effectively decouples the variables for stress consideration from the covariates/predictors that work for default prediction.

Now pay attention to the subscript for the macroeconomic stress variables on the right side of the equation, then, look at the subscript for the change in default covariates/predictors on the left side. Notice that both have the subscript “t”? This means that information pertaining to the macroeconomic stress variables enters into the default covariates over the same time period with no lag. In other words, the default covariates/predictors will respond at the same time, and we say that macroeconomic stress variables have a *contemporaneous effect* on the default covariates/predictors.

We take this opportunity to also point out the random error terms at the end of each equation. As much as we attempt to predict the impact which macroeconomic stress variables may have on the default covariates/predictors, there is an element of

uncertainty, and it is only wise to recognise it. Consequently, the model is not limited to predicting a single deterministic outcome under the prescribed macro scenario, but can predict a range of possible outcomes. Later on, we will use the above equation to simulate the system over the intended horizon of interest by drawing random numbers for the error terms.

We need to clarify further with regards to the second regression pertaining to changes in firm-specific attributes. Recall that we have a dataset for around 67,000 firms, and a good proportion of these do not have a long history of data for stable parameter estimation. Also, estimating the regression for each and every one of them might entail additional sampling errors which would be impounded into the final results. Even if we are only focusing on specific regions, the US has over 15,000 firms, while the Eurozone 12 is just shy of 12,000 firms and ASEAN 5 spills slightly over 6,000. To accommodate a large number of firms and for feasibility sake, we need some simplification and choose to conduct the stress testing regressions only for the *mean* of each firm-specific attribute ( $\bar{Y}_{i,j,t}$ ), where the *mean*<sup>12</sup> is taken across firms in each of ten industries.

To deal with individuality, the *mean* is subtracted from the individual firm's attribute values to obtain many residual series ( $Y_{i,j,t} - \bar{Y}_{i,j,t}$ ). This determines how far of an individual firm from its industry benchmark, i.e., a *relative position*. We then model each relative position series as an AR(3) process for the last 24 months prior to the start of a testing scenario.<sup>13</sup> However, missing values for some firm-specific variables will inevitably occur, causing the effective sample size to be unreasonably small. We thus adjust the autoregressive order to reflect the sample size. Specifically, the  $p$  in the AR( $p$ ) model takes a value of 3 if the sample size larger than 17, 2 if it is in the range of 12 to 17, 1 if it is in the range of 6 to 11, and 0 if it is below 6.

The stress-testing regression model for the mean and the residual's AR( $p$ ) dynamic are jointly employed to obtain future simulated values for each firm-specific attribute under the specified macroeconomic scenario. Apart from the statistical consideration explained above, the computational benefit arises from only conducting stress testing regressions for 10 series of cross-sectional mean values for each of the firm-specific attributes.

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<sup>12</sup> More precisely, we use the 20% trimmed mean, in other words, the top and bottom 20% of observations are dropped before taking the mean.

<sup>13</sup> See BuDA whitepaper v3.0 for the earlier version of the relative position regression

## Stress Variables

Table 6 below lists and briefly describes each of the standard variables implemented within BuDA. Within the implementation of BuDA, users may define their own macroeconomic stress variables, but for now, let us focus on the standard ones (i.e., default choice). BuDA provide the historical data for five country specific macroeconomic variables together with two common stress variables. In addition, CRI include country/industry level variables as CRI-PD predictors, these could be also considered as stress variables.

Type	Variables	Brief Description
<b>Country specific macro-economic variables</b>	GDP	Real Gross Domestic Product growth rate
	UNEMP	Difference of Unemployment rate
	CPI	Percentage change of consumer price index
	NEER	Percentage change of Nominal Effective Exchange Rate
	INT	Difference of 3-month interbank rate
	PPI	Property Price Index growth rate
<b>Country specific Common factors (CRI-PD predictors)</b>	Stock return	Monthly stock return
	Interest rate	3-month interbank rate (level)
	Aggregate DTD	Aggregate distance-to-default for financial and/or non-financial industry
<b>Other key stress variables</b>	GSCI	Percentage change of Standard and Poors Goldman Sachs Commodity Index
	VIX	Percentage change of the Chicago Board Options Exchange Volatility Index
<b>Credit Cycle Index</b>	CCI	Credit Cycle Index is provided by using aggregated CRI-PD. User can select the data from country to industry levels.

**Table 6: Selection of Stress Variables**

Real GDP reflects the state of an economy with its growth rate serving as a proxy for the growth in incomes and earnings of firms. A higher growth would generally lead to higher corporate earnings and lower default risk.

Unemployment rate affects the consumption and spending of households. An increase in the unemployment rate would generally result in lower revenues for firms, particularly those that are consumer-oriented (e.g., restaurants, retailers, etc.), and increase their default likelihoods.

Consumer price index provides a measure of inflation and controlling inflation rates is one of the primary objectives of monetary policy. High inflation is usually considered a signal

of macroeconomic mismanagement and a source of uncertainty. Higher inflation leads to increased costs and tends to impair credit quality. However, higher inflation may also reduce debt burden in real terms, and thereby improve creditworthiness.

Exchange rate affects the bottom-line of a wide range of firms directly or indirectly through trade and investment flows; for example, a stronger domestic currency will typically benefit importers and likely hurt exporters. Firms in a small open economy will be particularly sensitive to exchange rate movements. We use the BIS nominal effective exchange rate indices, which are calculated as geometric weighted averages of bilateral exchange rates.

Short-term floating rate is a common benchmark that banks use in determining lending rates for variable-rate loans. It is also the rate that is applicable to short-term corporate funding via debt/commercial paper markets. A higher borrowing rate/cost of funding is expected to increase default risk of firms owing to higher interest expense.

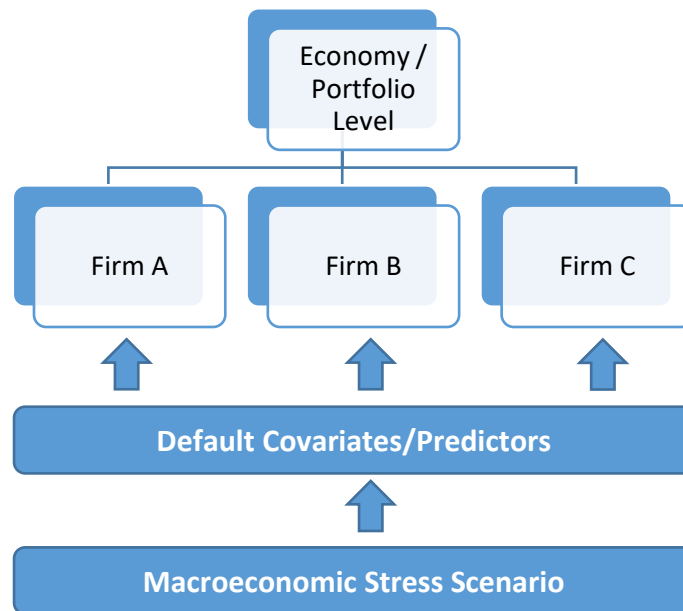
The S&P GSCI commodity index captures an important production factor cost. A higher commodity price typically benefits commodity producers but causes deteriorated creditworthiness for other companies.

The VIX index offered by the Chicago Board Options Exchange reflects the volatility in the S&P 500 index portfolio and is a commonly used proxy variable for gauging the risk level in US stock markets, but can also be indicative of how volatile stock markets in other economies, given the global linkage of financial markets and dominance of the US economy.

A word needs to be said about mixed-frequency data. GDP and unemployment data are typically available quarterly, whereas the other variables are available monthly. Furthermore, the PD data is available monthly (in fact daily) as well. Hence, if we simply treated the data as a monthly panel, GDP and unemployment would exhibit spikes every 3 months and flat at other times. This is not ideal. To address this issue, we incorporate two features. First, instead of jumps every 3 months, the data is interpolated so that it changes gradually rather than suddenly. Second, we apply a trick and rewrite the scenario analysis regressions in Equation 1 so that the variables become 12-month aggregates instead of 1-month. For those in the know, this is done by iteratively substituting the autoregressive formula into itself. The regression of default covariates/predictors against

stress variables on a 12-month aggregate basis makes the estimation much less sensitive to how the quarterly data are converted into monthly data.

### Top-down vs Bottom-up



**Figure 8: Bottom-up Aggregation**

As the name suggests, “bottom-up” refers to an approach where the impact on each individual firm in the pool of firms can be evaluated, and aggregated up into the portfolio or economy level. The methodology computes PD impacts for the most granular firm level, and builds it up to the overall portfolio level by aggregation or taking a statistic such as the average, median or percentile.

On the other hand, “top-down” scenario analysis involves first consolidating the data, and then performing the scenario analysis on the consolidated data. In a typical setup, a regulator or financial stability governing authority might not be able to work with granular individual firm data, hence limited to using consolidated data. The BuDA approach which leverages on the vast PD dataset produced by CRI provides a practical bottom-up solution.

## Stress Testing Scenarios

BuDA allows a range of macroeconomic stress scenarios to be specified without imposing too much limitation. One important point to note is that the stress scenario may be multi-period, occurring over several months or years instead of a single time period – in fact, the worst situations are likely an accumulation of undesirable events instead of a drastic single period event (although that can be accommodated).

Next, stress scenarios need not be stylised as a unidirectional shock. For example, a “V-shaped” recession and recovery used by the IMF may be applied. In this case, both the fall of GDP into recession and subsequent recovery are incorporated. Alternatively, a protracted recovery situation may also be analysed. As can be seen, the flexibility here allows even the actual evolution of historical events to form the stress scenarios.

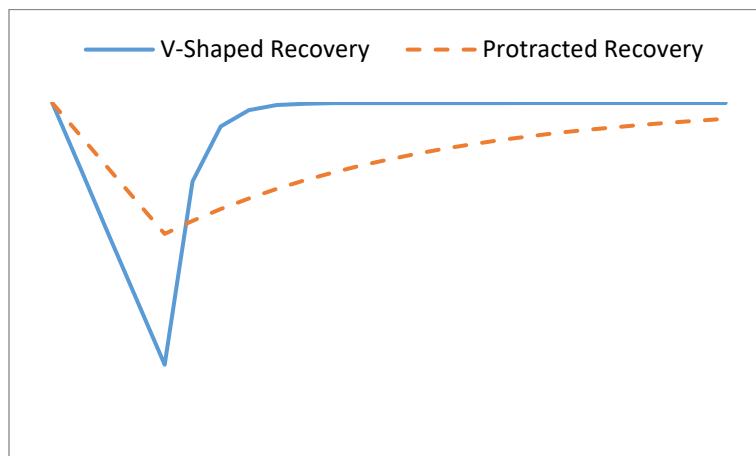


Figure 9: Examples of GDP Growth Rates under V-Shaped Recovery and Protracted Recovery



### III. BACKTESTING PERFORMANCE

How well does this scenario analysis methodology work? To answer this question, we can conduct backtesting and assess the suitability along a few dimensions. Does the methodology perform well for different regions or industries? Backtesting performance evaluation will answer these crucial questions. In the backtesting exercise, we simply go back in time and estimate the stressed PD from the actual realization of the stress variables. In doing so, it allows us to compare the stressed PD with the actual PD. To illustrate this, we provide two examples with the data from different regions and industries.

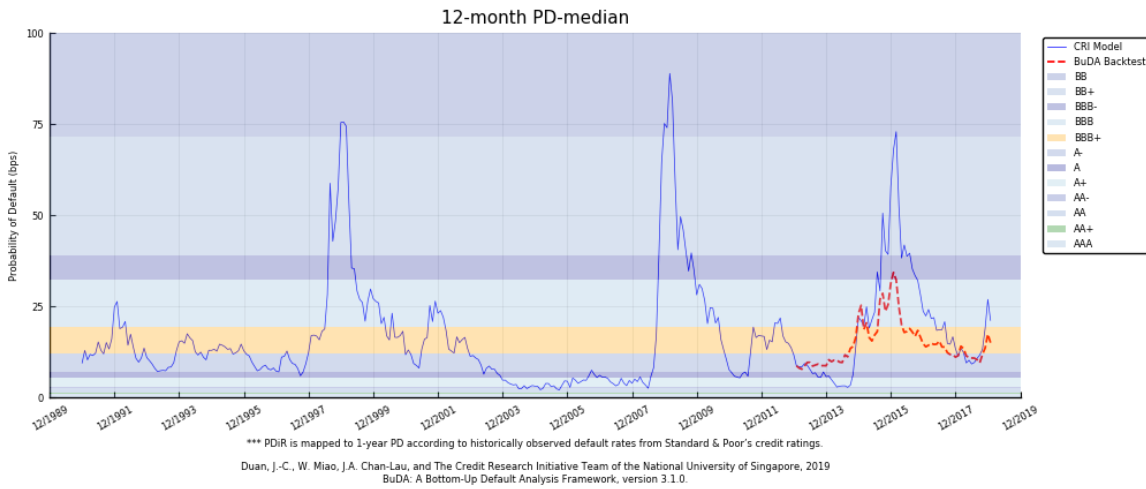
First, we consider the energy industry of US, where the stress variables are US-GDP, VIX and the percentage change of the crude oil price (OIL). Assuming we were back in time to Dec 2013 and interested in how the model performs 5 years later. Does it predict the new PD profile well? Since we are future beings, we already know what will happen to the stress variables which we specify as US-GDP, VIX and the percentage change of the crude oil price (OIL). We also know what will happen to the PDs over the same period. To do the test, the stress testing regression is estimated using the training data between Jan 1990 and Dec 2013. Then, the testing scenario is the timeseries of US-GDP, VIX and OIL from Jan 2013 to Jan 2019 as the BuDA inputs and let the model forecast the PD profile for the future date. If the predicted PDs of the actual scenario are good, they should match well with the actual changes in the CRI-PD profile.

In general, the model simulates around 4 years of time periods from January 2014 to Jan 2019, given the actual stress scenario, and obtains a possible PD outcome as at 31 December 2013. This is done 1,000 times to obtain 1,000 possible realizations of the PD profile. The averages of the simulated and actual PD profiles are then compared.

Our result<sup>14</sup> in Figure 10 shows that the stressed PD behave similarly to the actual PD. The stressed PD has an upward trend before hit the peak in Jan 2014 before the stress PD starts decreasing. This movement seems to fit the pattern of the actual data very well.

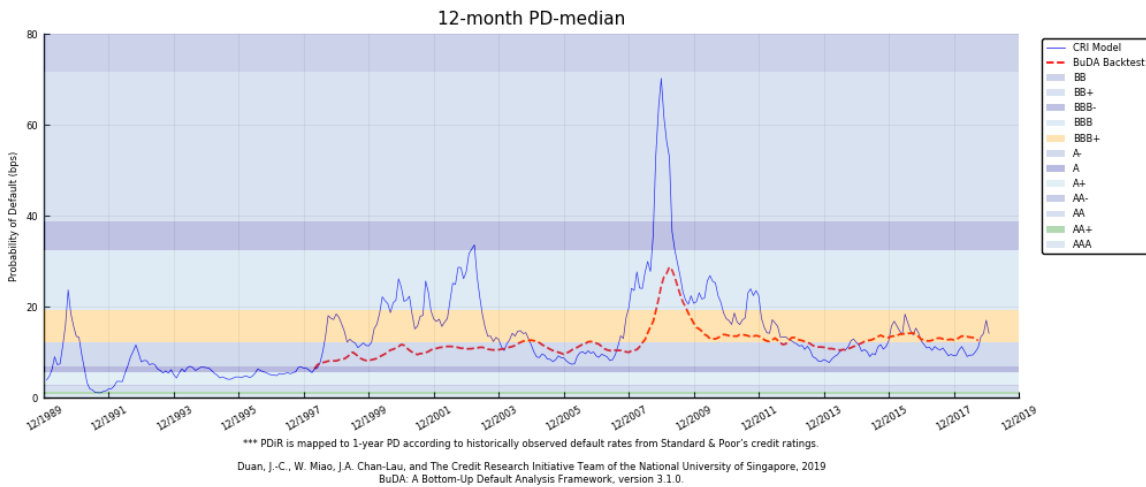
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<sup>14</sup> The result is generated by BuDA v3.1.0 toolkit



**Figure 10: Backtesting Performance for the Energy Industry in US**

In the second example, we consider the backtesting performance for entire UK market. However, rather than using the partial information of the historical data, the stress testing regression for UK was estimated using the whole sample period (i.e. from Dec 1989 to Sep 2018) with three stress variables (GDP, CPI and UNEMP). Then, the stress scenario is defined as the actual timeseries of those three macroeconomic variables between April 1998 and Sep 2018. The result shows that the stressed PD (red dash line) can mostly capture the trend/movement of the actual PD very well, especially from 2000 onward.



**Figure 11: Backtesting Performance for the UK Economy**

## IV. SUMMARY

The BuDA toolkit is built for the purposes of credit stress testing and macroeconomic scenario analysis, which is to investigate how firm PDs change under prescribed scenarios of interest. This can be useful to regulatory authorities, central banks, and commercial /investment banks for increased awareness and understanding of risks.

To estimate PDs under stress, BuDA provides a unique framework to assess the impact of macroeconomic stress variables on individual firms through three economy-wide common risk factors and ten firm-specific attributes. With the functional relationship in place, one can simulate values for all covariates (common and firm-specific) under the prescribed scenarios, and apply them to estimate the stressed PDs using the CRI-PD model. The simulated stressed credit risk profile for the portfolio of interest can then be constructed by a bottom-up aggregation.

## ABOUT THE CREDIT RESEARCH INITIATIVE

The Credit Research Initiative (CRI) was launched by Professor Jin-Chuan Duan in July 2009 at the Risk Management Institute of the National University of Singapore. Aiming at “Transforming Big Data into Smart Data”, the CRI covers over 68,000 public firms and produces daily updated Probabilities of Default (1-month to 5-year horizon), Actuarial Spreads (1-year to 5-year contract) and Probability of Default implied Ratings on over 34,000 currently active, exchange-listed firms in 128 economies. The CRI also distributes historical time series of over 34,000 inactive firms due to bankruptcy, corporate consolidation or delisting for other reasons. In addition, the CRI produces and maintains Corporate Vulnerability Indices (CVI), which can be viewed as stress indicators, measuring credit risk in economies, regions and special portfolios.

As a further step, the CRI converts smart data to actionable data to meet the customized demands of its users and offers bespoke credit risk solutions leveraging on its expertise in credit risk analytics. A concrete example is our development of the BuDA (Bottom-up Default Analysis) toolkit in collaboration with the IMF. BuDA is an automated analytic tool based on the CRI PD system, enabling IMF economists to conduct scenarios analyses for the macro-financial linkage.

The CRI publishes Weekly Credit Brief and Quarterly Credit Report, highlighting key credit-related events, offering insights based on the CRI PD of the entities involved, and providing useful statistics on credit risk of economies and sectors.

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